The indirect methods of control the output coordinates for the three-phase asynchronous electric motor

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***Abstract*— The article gives the mathematical description of an asynchronous motor with the indirect control of the output mechanical variables of an asynchronous motor in the electric drive. To determine the electromagnetic torque and angular velocity of the asynchronous motor in the electric drive the mathematical description is used in which the values are determined by the readings of the motor and easily measured values by means of known in practice devices. The proposed in the article the mathematical description for the indirect measuring the electromagnetic torque and angular velocity of the asynchronous motor in the electric drive does not contain the integral components that introduce the great error into the value of the controlled electromagnetic torque and angular velocity**.

Keywords*— *asynchronous electric motor, angular speed of rotation, torque, indirect control

I. INTRODUCTION

At present, during elaboration, testing and exploitation of the electric motors it is necessary to measure and register the output parameters such as the rates of rotation and moment on the roller. The elaborated methods and devices for measuring these parameters don’t meet the requirements. They are constructed on the bases of supplementary installed or connected with the electric motor micromachines of direct-current or alternating-current supply [4, 6, 9, 10]. These micromachines are very expensive and their design is complicated. Moreover their careful mechanical coupling with rotation parts of the electric motor is required.

Therefore in a number of cases their application is inexpedient due to their design or cost constraints. This leads to the necessity of elaboration, creation and application of other means for control of coordinates of the technological machinery with small content of material and power resources.

Therefore, it is actual to use other means of mechanical coordinates monitoring of general industrial devices, such as indirect control devices, when they are applied, these disadvantages are eliminated [2, 5, 7, 8].

The methods for the measuring the angular velocity of rotation and twisted moment of the three-phase asynchronous electric motor were designed. They are easy to use and provide the required accuracy of the result.

II. PROBLEM STATEMENT

In this regard, the objective of the research is to consider the options for devices of indirect control of coordinates - the angular velocity of rotation and torque of a three-phase squirrel-cage asynchronous motor. Moreover, unlike the existing devices, in the proposed ones the control is simplified, and there is a possibility of accounting for temperature of conductors of a stator winding, the frequency of a fundamental harmonic of the stator voltage, temperature coefficient, which depends on the conductor material, that improves the accuracy of the coordinates of the three-phase asynchronous motor.

III. MATHEMATICAL MODEL

Mathematical model to the angular velocity of rotation $\omega(t)$ of a three-phase squirrel-cage motor describes the equation

$$\omega(t) = \omega_{mes}(t) \cdot \lfloor 1 + \Delta\omega_{\text{int}}(t) + \Delta\omega_{dif}(t) \rfloor,$$

where

$$\omega_{mes}(t) = \sqrt{3} \cdot [u_a(t) - (z + R'_{r\alpha}) \cdot i_a(t)] /$$
$$/ (\int_0^{1/f} (z \cdot [i_a(t) + 2 \cdot i_b(t)] - [u_a(t) + 2 \cdot u_b(t)]) dt -$$
$$- L_\beta \cdot [i_a(t) + i_b(t)])$$

the measured value of the angular velocity;

$$\Delta\omega_{\text{int}}(t) = \frac{\int_0^{1/f} [u_a(t) - z \cdot i_a(t)] dt}{T_r^{'} \cdot [u_a(t) - (z + R'_{r\alpha}) \cdot i_a(t)]} \text{ -}$$

the dynamic integral component of the relative value of the angular velocity;

$$\Delta\omega_{dif}(t)=\frac{L_{\beta}\,\dfrac{di_a(t)}{dt}}{u_a(t)-(z+R'_{r\alpha})\cdot i_a(t)}$$ -

the dynamic differential component of the relative value of the angular velocity;

i_a, i_b, u_a, u_b – currents and voltage of correspondently phases A and B of the stator winding;

f – the frequency of a fundamental harmonic of the voltage supply of an asynchronous motor;

$R'_{r\alpha}=R'_r\cdot\alpha$ - active reduced resistance of stator winding taking into account α;

where R'_r - active reduced resistance of a stator;

$\alpha=\dfrac{L_S}{L'_r}$ - the coefficient equals the ratio of total inductance of the stator winding L_s to reduced total inductance of the stator winding L'_r;

$L_{\beta}=L_{\mu}\cdot\beta-L_s$ - the inductance taking into account the coefficient β;

Where L_{μ} - mutual inductance of stator and rotor windings;

$\beta=\dfrac{L_{\mu}}{L'_r}$ - the coefficient is equal to the ratio of mutual inductance L_{μ} to the reduced total inductance of the rotor winding L'_r;

$T'_r=\dfrac{L'_r}{R'_r}$ - the rotor time constant.

Active resistance of the stator winding taking into account the temperature coefficient α_t is determined by the following expression:

$$z=R_s\cdot\left[1+\alpha_t\cdot(t_{con}-20)\right];$$

where R_s - active resistance of the stator winding;

t_{con} - the temperature of conductors of the stator winding of the asynchronous motor.

Mathematical model to the value of the torque of a three-phase squirrel-cage asynchronous motor describes the equation

$$M(t)=\sqrt{3}\cdot p_n\cdot(i_a(t)\cdot\int_0^{1/f}\left[u_b(t)-z\cdot i_b(t)\right]dt-,$$
$$-i_b(t)\cdot\int_0^{1/f}\left[u_a(t)-z\cdot i_a(t)\right]dt)$$

where p_n – the number of pairs of motor poles.

IV. RESEARCH METHODS

To determine the electromagnetic torque and the angular velocity of the asynchronous motor the mathematical description is used, where in their values are determined according to the readings of the motor and easily measurable values using the known in practice devices.

$$\begin{cases}M(t)=\sqrt{3}\cdot p_n\cdot(i_a(t)\cdot\int\left[u_b(t)-R_s\cdot i_b(t)\right]dt-\\-i_b(t)\cdot\int\left[u_a(t)-R_s\cdot i_a(t)\right]dt),\\\omega(t)=\omega_m(t)\cdot\left[1+\Delta\omega_{int}(t)+\Delta\omega_{dif}(t)\right],\end{cases}\qquad(1)$$

The presence of the integral components available in the mathematical description of the asynchronous motor with the indirect control of the system of equations (1), leads to the errors, when determining the values of the currents and voltages, and can lead to the accumulation of significant errors in the values of the controlled electromagnetic torque and angular velocity of the asynchronous motor. Therefore, in the calculation of the integral components of the equation system (1) we use the known mathematical expressions for the analytic signals

$$u_a(t)=\sum_{k=0}^{\infty}A_k\cdot\sin(k\omega t+\varphi_k)+j\cdot\sum_{k=0}^{\infty}A_k\cdot\cos(k\omega t+\varphi_k),$$

Then,

$$\int_0^{1/f}[u_a(t)-z\cdot i_a(t)]dt=\int_0^{1/f}\tilde{U}_a(t)dt=$$
$$=\int_0^{1/f}\{\sum_{k=0}^{\infty}A_k\cdot(\sin k\omega^{*}t+\varphi_k^{*}+j\cdot\cos k\omega^{**}t+j\cdot\varphi_k^{**})\}dt-$$
$$-\int_0^{1/f}\{\sum_{k=0}^{\infty}B_k\cdot\sin k\omega^{**}t+\varphi_k^{**}+j\cdot\cos k\omega_k^{**}t+\varphi_k^{**}\}dt.$$

Using Euler's formula we can come from the trigonometric functions to the exponential ones [1, 3].

Then in general terms, we get that the integrand has the form

$$\tilde{U}_a(t)=\sum_{k=-\infty}^{+\infty}C_k\cdot e^{\frac{j\pi kt}{l}},$$

where the coefficient

$$C_k=\frac{1}{2l}\int_{-l}^{l}u(t)\cdot e^{-\frac{j\pi kt}{l}}dt,$$

i.e. the harmonics of e^{jkt} type give the range of wave numbers with the amplitude

$$C_k = \frac{1}{2l} \int_{-l}^{l} u(t) \cdot e^{-\frac{j\pi kt}{l}} dt.$$

Further it is possible to use the Fourier transform.

To the signal

$$\tilde{U}_a(t) = u(t) \cdot e^{jktn}.$$

$$k = -\frac{\pi}{l}.$$

Fourier transform has the form

$$\tilde{U}(j\omega) = u \int_0^{\infty} e^{-(a+j\omega)t} dt = \frac{u}{a + j\omega} = u(\omega) \cdot e^{j\varphi(\omega)}.$$

As the function is determined only to $t \succ 0$, one can use also sine and cosine of Fourier transform.

The approach of Fourier integral is usually given in a form

$$u(t) \approx \int_0^N (a(\omega)\cos\omega t + b(\omega)\sin\omega t)dt =$$

$$= \sum_{k=0}^{\infty} (C_k e^{-kt\omega} + C_{-k} e^{kt\omega}) = \sum_{k=0}^{\infty} -C_k^* \sin\omega t.$$

Thus, we get that the integral component goes into its orthogonal pair, representing the sum of the series.

The calculations of C_k coefficients are made according to the formulas presented in [1, 3, 11].

$$C_k = \frac{1}{2l} \int_{-l}^{l} u(t) \cdot e^{-\frac{j\pi kt}{l}} dt,$$

Using Parseval's formula and the inversion we get

$f(t) = e^{-\frac{j\pi kt}{l}}$, denoting $-\frac{j\pi kt}{l} = a$, we get

$$f(t) = e^{-at},$$

$$u(t) = \frac{1}{a + j\omega} = u(\omega) \cdot e^{-j\varphi(t)}.$$

$$g(t) = 1, \quad x = x^0 = 1, \quad \nu = 1$$

$$\int_{+0}^{l} u(t) \cdot e^{-at} dt = \frac{2^0 \cdot \Gamma(0 + \frac{1}{2})}{\sqrt{\pi} \cdot (a^2 + 1)^{0+\frac{1}{2}}} = \frac{\Gamma(\frac{1}{2})}{\sqrt{\pi} \cdot (a^2 + 1)^{\frac{1}{2}}}.$$

$$C_k = \frac{1}{2l} \cdot \frac{\Gamma(\frac{1}{2}) \cdot l}{\sqrt{\pi} \cdot (\pi^2 j^2 k^2 + l^2)} = \frac{\Gamma(\frac{1}{2})}{2\sqrt{\pi} \cdot (\pi^2 j^2 k^2 + l^2)}.$$

$$k = -\frac{\pi}{l}; \quad l = 1; \quad t = 1; k = -\pi.$$

From the above it follows

$$M(t) = \sqrt{3} \cdot p_n \cdot \left[i_a(t) \cdot \int \left[u_b(t) - R_s \cdot i_b(t) \right] dt - i_b(t) \cdot \int \left[u_a(t) - R_s \cdot i_a(t) \right] dt \right] =$$

$$= \sqrt{3} \cdot p_n \cdot i_a(t) \cdot (\left\{ \frac{\sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{1 - \pi^4}} - R_s \cdot \frac{\sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{1 - \pi^4}} \right\} -$$

$$- i_b(t) \cdot \left\{ \frac{\sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{1 - \frac{\pi^4}{4}}} - R_s \cdot \frac{\sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{1 - \frac{\pi^4}{4}}} \right\}) =$$

$$= \sqrt{3} \cdot p_n \cdot \left[(1 - R_s) \cdot e^{j\frac{\pi}{2}} \cdot \left\{ 2 \cdot i_b(t) \cdot \frac{1}{\sqrt{\pi^4 - 4}} - i_a(t) \cdot \frac{1}{\sqrt{\pi^4 - 1}} \right\} \right].$$

For $\omega_m(t)$

$$\int (R_s \cdot [i_a(t) + 2 \cdot i_b(t)] - [u_a(t) + 2 \cdot u_b(t)])dt =$$

$$= (R_s - 1) \cdot \left\{ \frac{1}{\sqrt{1 - \pi^4}} - \frac{2}{\sqrt{4 - \pi^4}} \right\} =$$

$$= (R_s - 1) \cdot e^{j\frac{\pi}{2}} \cdot \left\{ \frac{2}{\sqrt{\pi^4 - 1}} - \frac{1}{\sqrt{\pi^4 - 4}} \right\}$$

$$\omega_m(t) = \frac{\sqrt{3} \cdot [u_a(t) - (R_s + R'_{r\alpha}) \cdot i_a(t)]}{(R_s - 1) \cdot e^{j\frac{\pi}{2}} \cdot \left\{ \frac{2}{\sqrt{\pi^4 - 1}} - \frac{1}{\sqrt{\pi^4 - 4}} \right\} - L_\beta \cdot [i_a(t) + i_b(t)]}.$$

For $\Delta\omega_{int}(t)$

$$\int \left[u_a(t) - R_s \cdot i_a(t) \right] dt = \frac{1}{\sqrt{1 - \pi^4}} - \frac{2R_s}{\sqrt{4 - \pi^4}} =$$

$$= e^{j\frac{\pi}{2}} \cdot \left\{ \frac{2R_s}{\sqrt{\pi^4 - 1}} - \frac{1}{\sqrt{\pi^4 - 4}} \right\}$$

$$\Delta\omega_{int}(t) = \frac{e^{j\frac{\pi}{2}} \cdot \left\{ \frac{2R_s}{\sqrt{\pi^4 - 1}} - \frac{1}{\sqrt{\pi^4 - 4}} \right\}}{T_r' \cdot [u_a(t) - (R_s + R'_{r\alpha}) \cdot i_a(t)]}.$$

V. RESULTS

Thus, in its final form with account of the above-mentioned the mathematical description of the asynchronous motor with the indirect control of the electromagnetic torque and angular velocity can be written as follows

$$\begin{cases} M(t) = \sqrt{3} \cdot p_n \cdot \left[(1 - R_s) \cdot e^{j\frac{\pi}{2}} \cdot \left\{ 2 \cdot i_b(t) \cdot \frac{1}{\sqrt{\pi^4 - 4}} - i_a(t) \cdot \frac{1}{\sqrt{\pi^4 - 1}} \right\} \right] \\ \omega(t) = \omega_m(t) \cdot \left[1 + \Delta\omega_{int}(t) + \Delta\omega_{dif}(t) \right] \end{cases}, (2)$$

where

$$\omega_m(t) = \frac{\sqrt{3} \cdot [u_a(t) - (R_s + R'_{r\alpha}) \cdot i_a(t)]}{(R_s - 1) \cdot e^{j\frac{\pi}{2}} \cdot \left\{ \frac{2}{\sqrt{\pi^4 - 1}} - \frac{1}{\sqrt{\pi^4 - 4}} \right\} - L_\beta \cdot [i_a(t) + i_b(t)]};$$

$$\Delta\omega_{int}(t) = \frac{e^{j\frac{\pi}{2}} \cdot \left\{ \frac{2R_s}{\sqrt{\pi^4 - 1}} - \frac{1}{\sqrt{\pi^4 - 4}} \right\}}{T'_r \cdot [u_a(t) - (R_s + R'_{r\alpha}) \cdot i_a(t)]}; ;$$

$$\Delta\omega_{dif}(t) = \frac{L_\beta \frac{di_a(t)}{dt}}{u_a(t) - (R_s + R'_{r\alpha}) \cdot i_a(t)}.$$

VI. CONCLUSION

As a result, it is determined that the proposed analytical dependencies of the mathematical description of the asynchronous motor with the indirect control of the output mechanical variables allow to calculate the values and continuous monitoring of the electromagnetic torque and angular velocity in the electric drive.

To improve the quality of control of the output mechanical variables it is proposed to eliminate the integral component by the transition to an orthogonal pair using Fourier, Euler and Parseval transformation in the mathematical description of the asynchronous motor with the indirect control of the electromagnetic torque and angular velocity of the asynchronous motor.

The conducted research proves that the devices proposed in the paper have the improved accuracy of control of output coordinates of a squirrel-cage asynchronous motor in the dynamic working modes of an electric drive and a simpler implementation.

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